

# Notes on Fitting to a Line Or Challenges to the Square Root of N

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A while back, I pursued some research into sub-pixel resolution and obtainable accuracy. Although the work provided some interesting insights, it was never published. It is mostly theoretical with substantial Monte-Carlo simulation. It is an unfinished work. There is still more to learn from a theoretical standpoint as well as seeking correlation from real-world imaging.

For interest, I present a short synopsis of the results below.

## Introduction

It has long been assumed that a basic principle of statistics holds for sub-pixel work in image analysis. The principle states that if an observer makes  $N$  independent observations of a certain condition each observation having an uncertainty of  $\sigma$  then the average of those observations will have an uncertainty of  $\sigma/N^{1/2}$ .

Practical experience seems to show this principle holds to some degree in image processing, but that other effects seem to limit what can be obtained in practice. This work addressed one aspect of the limitation.

## Rational

Assume a regular square grid, a noiseless binary image, and a line (edge) running at exactly 45 degrees. It should be apparent that the line can have only discrete positions in the binary image.

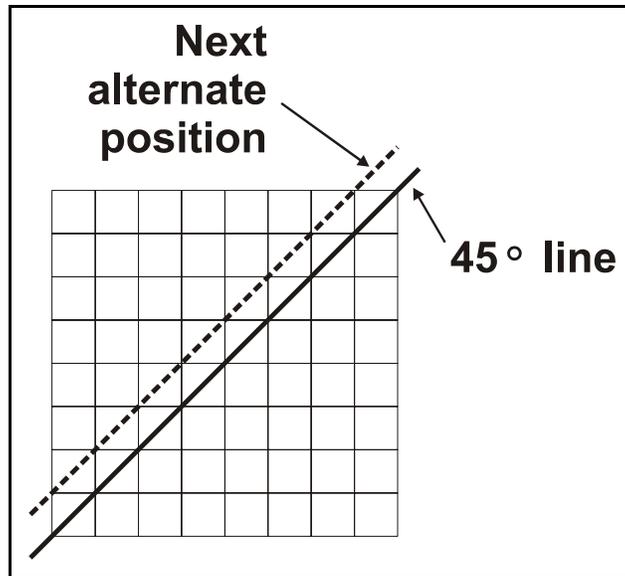


Figure 1 -- Allowable positions for a line (edge) in a binary image

The distance between these two lines is .707 pixel. The potential error is one-half this value or .354 pixel. The expression for the potential error of an edge in a binary image can be generalized to:

$$\varepsilon = \frac{\cos(\alpha)}{2R}$$

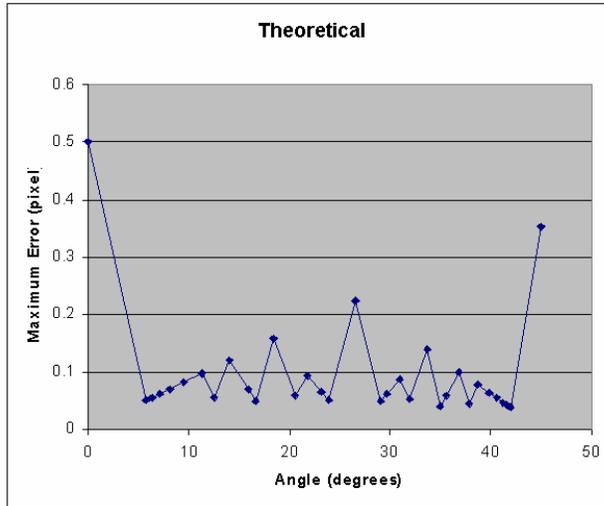
Where:

$\varepsilon$  is the maximum absolute error in line position

$\alpha$  is the angle of the line with the horizontal

R is the run length defined as the shorter of the length of the line or the shortest horizontal integer distance (in pixels) for a vertical integer distance

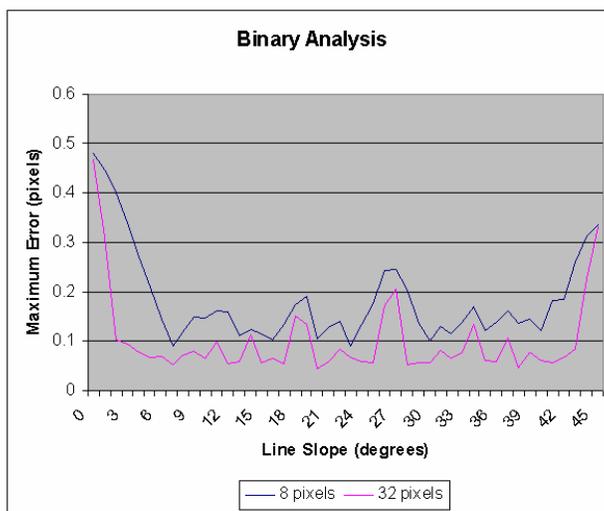
A graph of this function for horizontal distances up to 10 pixels is shown in Figure 2



**Figure 2 -- Theoretical binary edge maximum error**

These results are graphed only through 45 degrees. The range from 45 to 90 degrees would be a mirror image of these results, and contribute no additional information. The largest error occurs at an edge angle of zero degrees; a rise of 0 pixels for a run of 1 pixel. The second largest error occurs at 45 degrees; a rise of 1 pixel for a run of 1 pixel. The third largest error occurs around 26 degrees for a rise of 1 pixel for a run of 2 pixels. The error pattern continues to be reduced for other angles.

The results of a Monte Carlo simulation shown in Figure 3 helps to support the theoretical result. This simulation allows the slope of the edge, the noise in the image, and the fill factor of the image sensor to be varied. For the results shown below, the slope was set to a Gaussian distribution with a standard deviation of 1 pixel, there was no noise, and the fill factor was set to 1 (considered optimum).



**Figure 3 -- Simulation results for a binary edge**

The results of the simulation correlate very well with the theoretical expectations. Longer edges can give better results, but not by a function of the square root of  $N$ . Also, at critical angles, the maximum error is independent of edge length.

A similar simulation was performed using gray-scale analysis. The simulation used the same parameters as used for the binary image, and the edge was digitized to 256 gray levels. The edge detection technique was the zero crossing of the image convolved with a difference of Gaussians after Marr and Hildreth. The radii, in pixels, for the difference of Gaussian edge were 2 and 4. The results are shown below in Figure 4.

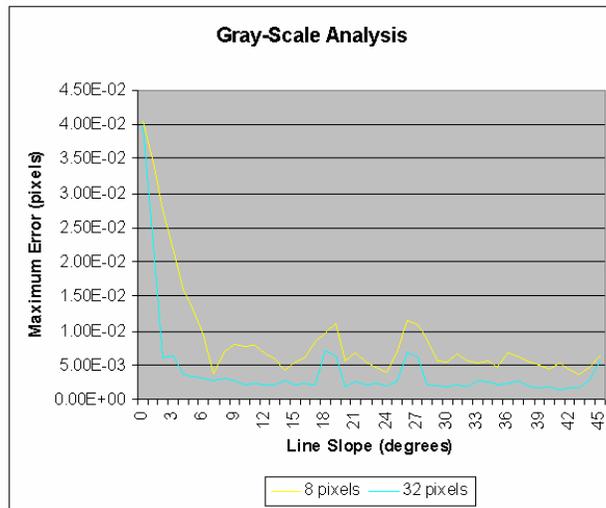


Figure 4 -- Simulation results for a gray-scale edge

These results show a much lower error in edge position than the binary analysis, but still have the same characteristics that certain critical angles give a poorer result.

## Conclusion

Whether edge detection and analysis is performed in binary or gray-scale, there are certain angles that give poorer results than others. The common practice in machine vision of aligning edges to be horizontal or vertical in the image gives the poorest possible results. Edges should be aligned several degrees off the horizontal for best results.

This study needs to be extended to include considerations of noise and fill factor in measurement precision. It also needs to be verified by practical experiments on real hardware.